Function Transformations 2

In this exercise, we explore horizontal and vertical stretches and compressions.

Please download the file `FunctionTransformations2.ggb` from my GeoGebra directory. Launch GeoGebra and open that file.

At the top of the drawing pad, you should see a slider showing "Page=1". Below that, find a pair of equations for line n, which is graphed nearby.

1) What two equations for line n are shown in the drawing pad? Write them down.

2) Move A to (2,1) and then (3,1). Notice that the blue rectangle is being *horizontally stretched*. How does the first equation change in response to that stretch?

3) Predict what the first equation will become when you move A to (5,1), and write down your prediction. Move A to (5,1). Were you right?

4) Move A back to (1,1). Now, move A to (1,2) and then (1,3). Notice that the blue rectangle is being *vertically stretched*. How does the first equation change?

5) Predict what the first equation will become when you move A to (1,5), and write down your prediction. Move A to (1,5). Were you right?

6) Predict what the first equation will become when you move A to (3,4), and write down your prediction. Move A to (3,4). Were you right?

7) Notice that the blue rectangle has been stretched both horizontally and vertically, by different amounts. Write a complete sentence to describe precisely how the original 1x1 square (gray) has been stretched in order to become this 3x4 rectangle.

8) Move A back to (1,1). Look at the picture, then move A to (2,2). What two equations for line n are shown in the drawing pad? Write them down.

9) How do your answers to #8 compare to your answers to #1? Why has one of the equations changed, while the other has not? Write at least two complete sentences.

10) Reset A to (1,1). If you were to vertically stretch the blue rectangle by a factor of 2—that is, double its height—where would A land? Move A to that point, and write down the two equations you see for line n.

11) Again, reset A to (1,1). This time, let's *horizontally compress* the blue rectangle. Let's halve its width. To do so, move A to (0.5,1), and write down the two equations you see for line n.

12) Compare your answers to #10 and #11. You should find that modifying the original line by a vertical stretch (factor 2) or a horizontal compression (factor $\frac{1}{2}$) will give us the same new line. Can you explain this?

Please move A back to (1,1). Advance the Page slider to Page=2. Now we're looking at a graph of $y = x^2$, and a pair of equivalent equations that describe it.

13) Write down the two equations that you see.

14) Move A to (2,1) and then (3,1). Notice that the blue rectangle is being *horizontally stretched*. How does the first equation change?

15) Predict what the first equation will become when you move A to (5,1), and write down your prediction. Move A to (5,1). Were you right?

16) Move A back to (1,1). Now, move A to (1,2) and then (1,3). Notice that the blue rectangle is being *vertically stretched*. How does the first equation change?

17) Predict what the first equation will become when you move A to (1,5), and write down your prediction. Move A to (1,5). Were you right?

18) Predict what the first equation will become when you move A to (3,4), and write down your prediction. Move A to (3,4). Were you right?

19) Reset A to (1,1). Apply a horizontal compression by moving A to (0.5,1). By what number was the width of the gray square multiplied, to produce the blue rectangle? ______ Write down both of the equations that you see. Show algebraically that the two equations are equivalent—that is, simplify one of them so that it becomes the other.

20) Move A to (1,4). What kind of transformation would turn the gray square into the blue rectangle, as it currently appears? ______________ ______________ by a factor of _______. In the space below, write both of the equations that you see. Algebraically show that they are equivalent to each other and to the equations from #19.

Let's collect some thoughts. A horizontal stretch or compression happens when "x" is replaced with "$\frac{x}{c}$", for some number c. For instance, replacing x with $\frac{x}{2}$ will produce a horizontal stretch by a factor of 2: it makes the graph twice as wide. Meanwhile, a vertical stretch or compression happens when we multiply the whole function by a number. For example, if we change $y = x$ into $y = \frac{1}{2}x$, we've multiplied the function by $\frac{1}{2}$—and made the graph half as tall.

A horizontal compression is not the same as a vertical stretch: we can see that they do different things to the blue rectangle. However, sometimes these *different transformations* can turn a given function into the *same transformed function*. Consider the two examples above. Starting with $y = x$, whether we double the width or halve the height, we get the same function in the end: $y = x/2$.

On the other hand: even though this pair of transformations did the same thing to $y = x$, they won't do the same thing to other functions. For instance, if we start with $y = x^2$, then "double the width" is represented by $y = (\frac{x}{2})^2$, which is equivalent to $y = \frac{x^2}{4}$, also known as "quarter the height". Likewise, "halve the width" is represented by $y = (\frac{x}{0.5})^2 = (2x)^2 = 4x^2$, which is also what you would get by quadrupling the height. I hope you found these equations in #19 and #20 above.

Move A to (1,1), and advance the Page slider to Page=3. This is the function $y = \sqrt{1 - x^2}$, the top half of the unit circle.

21) Can you find a vertical stretch or compression that transforms this function in the same way as some horizontal stretch or compression? Which transformations did you find—or, why couldn't you find any?

22) Move A to (-1,1). How would you describe this transformation?